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# The impact toughness of discontinuous boron-reinforced epoxy composites

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Previous theories for the impact strength of discontinuously-reinforced composites predict that the toughness is a maximum when critical transfer length fibres are used. Experiments utilizing mini-Charpy specimens of unidirectional boron-fibre-reinforced epoxy composites have been conducted which corroborate this prediction. However, calculations of the fracture energy, based on a uniform interfacial shear stress during fibre pull-out, proved inadequate for the reinforced epoxy composites. Revisions to existing theories are presented to take into account the non-uniformity of the interfacial shear stress distribution along the fibre length and catastrophic failure of the interfacial bond.

## Nomenclature

$A_f$ = fibre cross-sectional area
$E_f$ = fibre Young's modulus
$G_m$ = matrix shear modulus
$l$ = fibre length
$L$ = fibre pull-out length
$l_c$ = fibre critical length
$r$ = fibre radius
$R$ = half fibre centre-to-centre spacing
$V_f$ = fibre volume fraction
$W$ = mean work of fracture per unit area of specimen cross-section
$x$ = distance from fibre end
$y$ = dummy variable of integration
$\gamma$ = surface energy
$\epsilon$ = strain in composite
$\sigma$ = tensile stress on fibre
$\sigma_f$ = fibre fracture strength
$\tau$ = interfacial shear stress

## 1. Introduction

The toughness of a material is commonly defined as the energy absorbed during the propagation of a crack. In the manner of Orowan [1] and Irwin [2], $\gamma$, the surface energy, may be interpreted as the total work of fracture per unit area of fracture surface. In fibre-reinforced materials, the fracture surface area may be greatly increased by fibre pull-out and crack path deflection, which results in composites of higher toughness than either component alone.

As discussed by Cottrell [3] and Kelly [4, 5], the toughness of a composite material reinforced with short, discontinuous fibres may be maximized by the use of fibres of length on the order of the critical-transfer length. As the fibre length, $l$, decreases to the critical length, $l_c$, fracture increasingly occurs by fibre pull-out rather than by fibre fracture. At a fibre length just slightly less than $l_c$, fracture occurs exclusively by pull-out and the toughness is a maximum.

Assuming a constant interfacial shear stress, $\tau$, exerted between fibre and matrix during pull-out, Cottrell [3] predicts a work of fracture of,

$$2\gamma = \left(\frac{V_f\,\tau}{3r}\right)\left(\frac{L}{2}\right)^2 \tag{1}$$

where: $\gamma$ = surface energy; $V_f$ = volume fraction fibre; $\tau$ = interfacial shear stress; $r$ = fibre radius; $L$ = fibre pull-out length.

When the fibre length is equal to the critical length, $l_c$, Equation 1 becomes:

$$2\gamma = \left(\frac{V_f\,\tau}{3r}\right)\left(\frac{l_c}{2}\right)^2 = \frac{V_f}{12}\,\sigma_f\,l_c \tag{2}$$

where $\sigma_f$ = fibre strength.

It is the intent of the present work to expand the applicability of the above expression to include systems which fail under the action of *non-uniform* interfacial shear stresses. The analysis is compared with the results of Charpy-impact tests on discontinuous boron-fibre reinforced-epoxy composites.

## 2. Experiment

To check the validity of Equation 2 in a brittle-matrix, brittle-fibre composite, mini-Charpy impact specimens were prepared containing unidirectionally aligned discontinuous fibres in an epoxy matrix. One sample each containing 0.25, 0.50, 0.75 and 2.0 in. long boron fibres, 0.004 in. diameter, of $55 \times 10^6$ psi modulus, were made in silicone rubber moulds. Photo-stress L-08 epoxy resin (100 parts) was mixed with L-08 catalyst (12 parts) and stirred until viscous. The resin mixture was then heated to 27°C and cast over the fibres.

Fibre alignment was achieved by combing the fibres with a glass rod while the resin was still in a fluid condition. Good unidirectional alignment was readily attained with the longer fibres (Fig. 1). The 0.25 in. fibres however, required the partitioning of the mould lengthwise into four sections. Partitions were constructed using 0.001 in. thick mylar sheets. The 0.25 in. fibres were then treated as before, i.e. combed into unidirectional alignment and the mylar films carefully removed. All samples were allowed to cure at room temperature for 24 h. The unidirectional fibre alignment achieved for the shorter fibres in this manner is also shown in Fig. 1. A greater degree of misalignment is apparent in the samples containing the 0.25 in. fibres due to handling difficulties and the more complicated sample preparation technique. A fibre volume fraction of approximately 30 % was incorporated into all samples. Cured specimens were removed from the moulds and surface ground to the final dimensions of 0.197 in. thick by 0.394 in. wide by 2.165 in. long. A 45° notch of depth 0.079 in. and 0.010 in. radius was ground into the centre of the specimens.

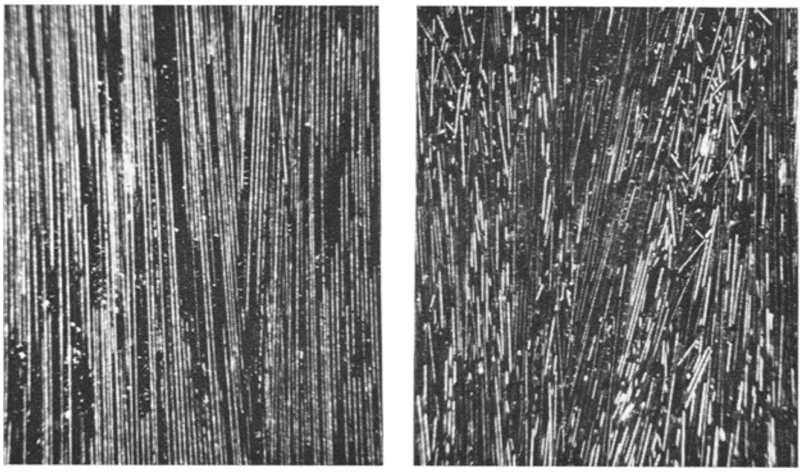
After machining and measurement of final specimen dimensions, the samples were impact tested on a Manlabs Charpy machine model CIM-24. Impact test results are given in Table I.

TABLE I Mini-Charpy impact test results for boron epoxy

| Fibre length | Fracture energy (ft. lb) | Fracture energy (in. lb/in)$^2$ |
|---|---|---|
| 0 (control) | 0.03 | 6 |
| 1/4 | 0.70 | 133 |
| 1/2 | 2.70 | 515 |
| 3/4 | 1.35 | 257 |
| 2 | 0.77 | 147 |

The values for fracture energy of the longer fibre composites are in reasonable agreement with those obtained by other investigators [6], thus tending to verify the experimental technique.

To determine the boron-fibre volume fraction, the samples were sliced near the notch, potted in



(a) (b)

*Figure 1* Longitudinal section of mini-Charpy impact specimens revealing fibre alignment; (a) 2.0 in. fibres, (b) 0.25 in.

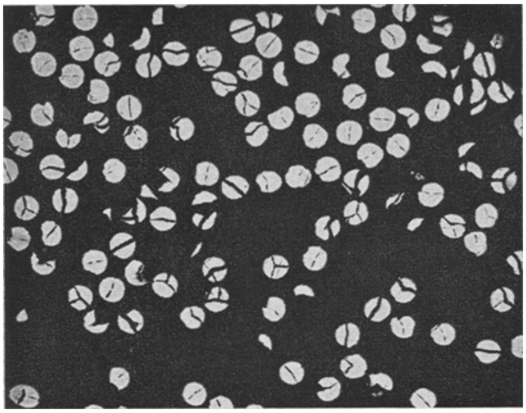


*Figure 2* Cross-section showing fibre distribution in an impact specimen containing 0.50 in. long, 0.004 in. diameter boron fibres.

epoxy, polished and examined metallographically. A sample cross-section showing a typical fibre distribution is shown in Fig. 2 for a specimen containing 0.50 in. long fibres. Fibre volume fractions ranged from 0.28 to 0.30.

Fracture surfaces were vapour-coated with gold and examined with the scanning electron microscope. In addition to fibre pull-out and brittle fracture of the epoxy matrix and boron fibres, the fracture surfaces revealed fibre-matrix debonding, as seen in Fig. 3.

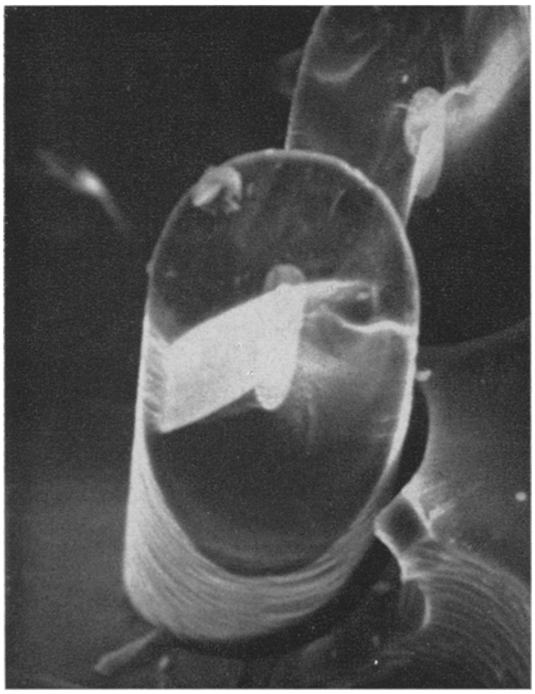


*Figure 3* Scanning electron micrograph of the fracture surface of a sample containing 0.25 in. long, 0.004 in. diameter boron fibres.

## 3. Results and discussion

A calculation of the fracture energy for this boron-epoxy composite as predicted by Equation 2, using an as-received fibre strength of 350000 psi (no fibre degradation from specimen fabrication was expected), exceeds the experimental results by an order of magnitude. Such a discrepancy suggests that the assumption of a constant interfacial shear stress, $\tau$, during pull-out may not be applicable in this brittle system loaded under impact conditions. For this particular boron-epoxy system, it has been observed that catastrophic brittle fracture of the fibre-matrix interface takes place [7]. Therefore, it may be assumed that when the peak value of the interfacial shear stress distribution reaches the interfacial shear strength, brittle failure of the fibre-matrix interface occurs. This type of behaviour has been discussed by Lawrence [8] and is also supported by the appearance of debonded interfaces in Fig. 3.

The shear stress distribution along the fibre-matrix interface is non-uniform, with stress concentrations at the fibre ends which decrease rapidly down the length of the fibre. This distribution, although only very approximately represented, has been estimated in closed form by Cox [9] for fibres of circular cross section in a hexagonal array, and may be represented as:

$$\tau(x) = E_f\,\epsilon \left[\left(\frac{G_m}{E_f 2 \ln \frac{R}{r}}\right)^{1/2} \left(\frac{\sinh \beta\left(\frac{l}{2} - x\right)}{\cosh \beta \frac{l}{2}}\right)\right] \quad \ldots\ldots (3)$$

$$\beta = \left[\frac{G_m}{E_f}\left(\frac{2\pi}{A_f \ln \frac{R}{r}}\right)\right]^{1/2}$$

where $E_f$ = fibre Young's modulus; $G_m$ = matrix shear modulus; $\epsilon$ = composite strain; $A_f$ = cross sectional fibre area; $l$ = fibre length; $r$ = fibre radius; $R$ = half-fibre centre-to-centre spacing; $x$ = distance from fibre end.

The non-uniform interfacial shear stress delineated by Equation 3 may be included in the derivation of Equation 2. The energy required to set up the interfacial shear stress distribution on a fibre which spans the crack plane and whose end is within a distance $x$ of the crack plane may then be represented as:

$$\pi r^2 \int_{y=0}^{x} \sigma \, dy = \pi r^2 \int_{y=0}^{x} \frac{2\tau(y)\, y \, dy}{r} \quad (4)$$

where $\sigma$ = tensile stress in the fibre and $y$ = dummy variable of integration.

The mean value of the energy per fibre required to stress a population of fibres, uniformly varying in length from zero to $l/2$ from the crack plane, is:

$$\int_{x=0}^{l/2} \int_{y=0}^{x} \frac{2\pi r \, \tau(y) \, y}{l/2} \, dy \, dx \, . \quad (5)$$

Equation 5 may be integrated by parts, using

$$u = \int_{y=0}^{x} \frac{2\pi r \, \tau(y) \, y}{l/2} \, dy$$

and $dv = dx$, which results in a mean value of the energy of:

$$\int_{x=0}^{l/2} \frac{4\pi r}{l} \left(\frac{l}{2} - x\right) \tau(x) \, x \, dx \, . \quad (6)$$

The mean work of fracture, per unit area of specimen cross-section, is obtained by multiplying Equation 6 by $V_f/\pi r^2$. For a composite reinforced with fibres of subcritical length in which all fibres will pull out, the mean work of fracture then becomes:

$$W = \frac{4V_f}{rl} \int_{x=0}^{l/2} \left(\frac{l}{2} - x\right) \tau(x) \, x \, dx \, , \text{ for } l \leq l_c \, . \quad \ldots\ldots (7)$$

When reinforcement is attained using fibres of length greater than $l_c$, a fraction $l_c/l$ of fibres will pull out rather than fracture, and the work of fracture becomes:

$$W = \frac{4V_f}{rl_c} \left(\frac{l_c}{l}\right) \int_{x=0}^{l_c/2} \left(\frac{l_c}{2} - x\right) \tau(x) \, x \, dx \, , \quad \text{for } l \geq l_c \, . \quad (8)$$

Under the assumption of catastrophic brittle fracture of the fibre-matrix interface when the peak interfacial shear stress reaches the strength of the interface, the work of fracture for subcritical aspect ratio fibres may be calculated from Equation 7 and for supercritical aspect ratio fibres from Equation 8.

In previous studies of this boron-epoxy system, a critical transfer length, $l_c$, of approximately 0.5 in. was determined [7], which agrees with the experimental results obtained in this paper (Table I). The interfacial shear strength was also determined for this system to be about 1750 psi [7].

For the present study it was assumed that, at a composite strain associated with an interfacial shear stress distribution which peaks at 1750 psi at the fibre ends, the fibre-matrix interface fails catastrophically. A centre-to-centre fibre spacing of 0.007 in. was used for the purposes of analysis, which corresponds to a fibre volume fraction of approximately 30% in a hexagonal array. Using the above assumptions together with the interfacial shear stress distribution of Cox, Equation 3, and the measured values of matrix shear modulus, $G_m$, of $1.67 \times 10^5$ psi and fibre modulus, $E_f$, of $55 \times 10^6$ psi, Equations 7 and 8 may be solved. The necessary integration was performed on a digital computer using a Simpson's rule method. Fracture energies predicted by Equations 7 and 8 are plotted as a function of fibre length and compared to the experimental results in Fig. 4.

The present analysis is in closer agreement with experiment than other treatments [3-5] which use a uniform interfacial shear stress and a pull-out mechanism. However, as may be seen in Fig. 4, Equations 7 and 8 are conservative in predicting the experimental fracture energies. The greatest limitation of the current analysis stems from the very approximate representation of the actual shear stress distribution along the fibres predicted by Equation 3 [9]. This equation for interfacial shear stress was used however, because it is a manageable closed form solution and facilitates a demonstration of the proposed mechanism. Other limitations are due to neglecting energy absorption from fibre and matrix fracture and fibre splitting (Fig. 2), although these contributions are probably minimal.

The magnitude of the calculated results and the observation of debonded fibres (Fig. 3) do suggest however, that the assumption of catastrophic failure of the interface is a valid mechanism, i.e. failure occurs catastrophically when the peak of the shear stress distribution reaches the strength of the interface. The maximum deviation between theory and experiment occurs at the critical fibre length, $l_c$, and suggests that an additional contribution to toughness is being made after interface failure but during fibre pull-out. Such a contribution could be due to a friction imposed between fibres and matrix by specimen bending during failure.

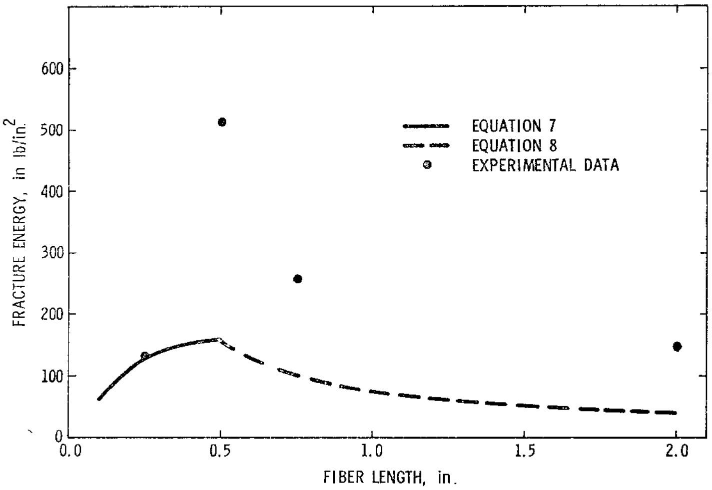



*Figure 4* Fracture energy versus fibre length.

This effect would be a maximum at $l_c$ where the greatest amount of pull-out takes place.

Equations 7 and 8 were examined to determine the sensitivity of toughness to various material parameters. The fibre volume fraction, $V_f$, and fibre spacing enter Equations 7 and 8 in such a way that increases in fibre content increase the amount of pull-out, but also reduce the fibre spacing causing higher shear stress concentrations. The net effect on toughness is minimal since these two factors tend to cancel each other. This prediction is contrary to what has been observed in *continuous* carbon-fibre composites where the impact energy tends to increase significantly with increasing fibre content [10]. The present analysis does not apply as $l$ approaches infinity, or more specifically, for continuous fibres where there are no fibre ends to cause stress concentrations. For continuous fibres the analysis would have to include the statistical variation of fibre strength and the idea of a critical flaw spacing [11, 12]. In addition, the assumption of catastrophic failure of the interface would be more tenuous.

In the limit as $l$ approaches zero, the toughness goes to zero when in fact it should approach the toughness of the matrix. This may easily be corrected by adding a rule of mixtures term to Equation 7. However, the toughness of the unreinforced matrix is negligible (Table I) and may be ignored for the purposes of this paper, as it has been in other studies [13].

## 4. Conclusions

The results of this study suggest that close control of fibre length is necessary to achieve maximum toughness in discontinuously-reinforced, brittle-fibre, brittle-matrix composites subjected to impact loading conditions. The Charpy impact toughness of the boron-fibre-reinforced epoxy system is a maximum for samples containing fibres close to the critical length to diameter ratio. Furthermore, in the boron-epoxy system subjected to impact loading, a constant or uniform interfacial shear stress does not appear to be maintained during fibre pull-out and should not be used in the analysis of toughness. Rather it is suggested that a non-uniform interfacial shear stress distribution be used to calculate toughness in composites where fracture is controlled by the interfacial strength. The proposed failure mechanism is that catastrophic fracture of the fibre-matrix interface is precipitated when the peak in the shear stress distribution reaches the interfacial strength at the ends of fibres.

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